Combined First and Second Semester B.Tech. Degree Examination, December 2015
(2013 Scheme)
13.101 : ENGINEERING MATHEMATICS – I (ABCEFHMNPRSTU)

Time : 3 Hours | Max. Marks : 100

PART – A

Answer **all** questions. **Each** question caries 4 marks.

1. Show that the curvature at any point on the rectangular hyperbola $xy = c^2$ is $\frac{2c^2}{\sqrt{x^2 + y^2}}$.

2. If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1 - x^2)\, y_{n+1} - (2n + 1)\, xy_n - n^2 y_{n-1} = 0$.

3. Evaluate $\iiint\limits_R (x + y + z)\; dx\, dy\, dz$ where R is given by $R = \{(x, y, z) : 0 \le x \le 1, 1 \le y \le 2, 2 \le z \le 3\}$.

4. Find $L(te^{-t} \cos t)$.

5. For the matrix $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$, obtain A^{-1}, using Cayley Hamilton Theorem.

PART – B

Answer **one full** question from **each** Module. **Each** question carries **20** marks.

Module – I

6. a) Find the value of the constants a and b in order that

$$\lim_{x \to 0}\left(\frac{x(1+a\cos x)-b\sin x}{x^3}\right)=1.$$

b) Find the evolute of the curve, $x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$.

7. a) Find $J = \frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$ if $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$.

b) Find the maxima and minima of $x^3y^2(12 - x - y)$ if $x > 0$, $y > 0$.

Module – II

8. a) Evaluate $\iint_A (x^2 + y)\,dx\,dy$ where A is the region in the positive quadrant for which $x + y \le 1$.

b) Evaluate the area bounded between the curves $y = x$ and $y^2 = 4x$.

9. a) Evaluate $\iint (x^2 + y^2)\,dx\,dy$ over the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

b) Evaluate $\iiint x^2 y\,dz\,dy\,dx$ throughout the volume bounded by the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ in the positive quadrant a, b, c > 0.

Module – III

10. a) Find the inverse Laplace transform of

i) $\frac{S}{(S-3)^5}$ ii) $\frac{3-2S}{(S-2)^2(S-1)^2}$

b) Using convolution theorem, find the inverse Laplace transform of $\frac{S}{(S^2+a^2)^2}$.

11. a) Solve the equation $(D^3 + 4D)\, y = \text{Sin } 2x$.

b) By using Laplace transforms solve $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - 5y = 5$ with $y = 0$, $\frac{dy}{dt} = 2$ when $t = 0$.

Module – IV

12. a) Find the eigen values and the eigen vectors of the matrix,

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & -1 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

b) Reduce $3x^2 + 5y^2 + 3z^2 - 2yz - 12zx - 2xy$ to canonical form. Specify the nature.

13. a) Find the values of λ and μ in order that the system of equations, $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$ and $2x + 3y + \lambda z = \mu$ has a unique solution.

b) Verify that the matrix, $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation and hence find A^4.
